4. Homework Assignments

Dynamical Systems II

Bernold Fiedler

http://dynamics.mi.fu-berlin.de/lectures/

due date: Thursday, November 13, 2014

Problem 1: [counts as 2 problems] Let a, b denote orientation-preserving $\mathcal{C}^2$-diffeomorphisms of S^1 and ϱ the rotation number. Prove or disprove:

(i) $\varrho(a) + \varrho(a^{-1}) = 0$;

(ii) $\varrho(aba^{-1}) + \varrho(b^{-1}) = 0$;

(iii) if $ab = ba$ and $\varrho(a) \notin \mathbb{Q}$ then $\varrho(ab) = \varrho(a) + \varrho(b)$;

(iv) if $\varrho(a) = \varrho(b) \notin \mathbb{Q}$ then $ab = ba$;

(v) if $\varrho(a) = \varrho(b) \notin \mathbb{Q}$ then $\varrho(ab) = 2\varrho(a)$;

(vi) $\varrho(ab) = \varrho(ba)$;

(vii) if $n \in \mathbb{Z}$ then $\varrho(a^n) = n\varrho(a)$;

(viii) $\varrho(ab) = \varrho(a) + \varrho(b)$.

Problem 2: Calculate the rotation number $\varrho(\alpha)$ of the time-2π-map of the differential equation

$$\dot{x} = \alpha + \cos(x - t), \qquad x \in S^1.$$

Problem 3: Consider the map $A : \mathbb{R} \to \mathbb{R}$,

$$A(y) \;=\; \begin{cases} 2y, & 0 \le y < 1 \\ A(y-1) + 1, & 1 \le y \\ A(y+1) - 1, & y < 0 \end{cases}$$

Thus $A(y+1) = A(y)+1$ for all y and A defines a map $\tilde{A} : S^1 \to S^1 = \mathbb{R}/\mathbb{Z}$. However A and $\tilde{A}$ are not homeomorphisms. Nonetheless, try to define the usual "rotation number" $\varrho(y_0)$ for initial conditions y_0. Does $\varrho(y_0)$ depend on y_0?